Lecture 3: Formal Proofs and Whatnot

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1 Whatnot

This lecture was very boring. Prof. Colding spent the whole time going over proofs which have already been proven in class and writing them down in a formal way. Is it useful to understand formal proofs? Yes. Should you spend 80 minutes going over proofs we have already seen and making them formal? No. As such, this document is very small. I don't think it is worth going over the proofs in the way he did.

There are three important things you must know.

1. Subsets and boundaries

   There is an important edge case when it comes to subsets and boundaries. If we have a set, $\mathbb{S} = (0, \infty)$, and another set, $\mathbb{A} \subseteq \mathbb{S}$, and $\mathbb{A} = (0, 1)$, then A is bounded from above. $\forall\ a \in \mathbb{A}, a < 1$. However, $\mathbb{A}$ is not bounded from below. This is because $\mathbb{A}$ is specifically a subset of $\mathbb{S}$, not $\mathbb{R}$. 0 and all of the negative numbers are not included in $\mathbb{S}$, and so there is no bound for the lower edge of $\mathbb{A}$.

2. Least upper bound operation

   $M_1 = \sup \mathbb{A}$ denotes a least upper bound

3. Greatest lower bound operation

   Up until now, we have not seen the concept of the greatest lower bound, but it is exactly as it sounds. $m_1 = \inf \mathbb{A}$ denotes the greatest lower bound, and m_1 will satisfy $m_1 \geq m$.

2 Formal Proofs

As mentioned above, I don't think it will be useful to go over the proofs themselves, but instead just mention a tip that Prof. Colder gave when making proofs.

"When writing a proof, it is important to say how you are going to prove it, (by induction, contradiction, etc) and then define all the terms you need to, which may seem to come from nowhere."

That is it.